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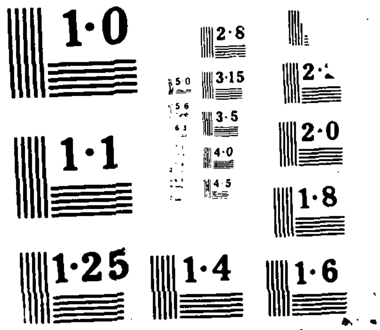
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Radiative Heat Transfer and Selective Emission in Planar Slab Coatings on Hot Emissive Substrates

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The spectral radiance of infrared radiation emitted from a smooth planar coating in thermal contact with a hot substrate is analyzed by radiative transport theory. The solution of the radiative transfer equation, which governs the spectral radiance within the coating, is used to determine whether selective (nonblackbody) emitter coatings in infrared banded regions can be obtained in the limiting cases of optically thin or thick regimes. For the reader not interested in the mathematical details of this report, the key conclusions are given following Eq. (22). The results are presented in a summary that shows how varying the optical thickness of the coating, the coating material, and the substrate parameters affect the emitted radiation.

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CONTENTS

"As the matter becomes clear, one tends to forget the difficulties and the mystery which surround the problem in the beginning. What once was strange becomes first evident and then natural! The problem ceases to be a problem; it fades into nothing and disappears in the night of the past."

V. Kourganoff

NOMENCLATURE

a	absorptivity
A	area, cm^2
c_p	specific heat at constant pressure, cal g^{-1} K^{-1}
d	layer thickness, cm
E_λ	radiative energy, cal
$\mathcal{J}_\lambda^e(T)$	spectral emission radiance, W cm^{-3} μm^{-1} sr^{-1}
k	thermal conductivity, cal cm^{-1} s^{-1} K^{-1}
N_λ	spectral radiance, W cm^{-2} sr^{-1} μm^{-1}
N_λ^b	spectral radiance of blackbody, W cm^{-2} sr^{-1} μm^{-1}
N_p	Number of particles per volume, cm^{-3}
$P(\hat{\Omega}' \cdot \hat{\Omega})$	phase function
q_c	heat flow owing to conduction, cal cm^{-2} s^{-1}
q_r	heat flow owing to radiation, cal cm^{-2} s^{-1}
r_λ	reflectivity
s	length, cm
S	energy generation rate per unit volume per unit time, cal cm^{-3} s^{-1}
T	absolute temperature, K
W_λ^b	Planck's function at wavelength λ, W cm^{-2} μm^{-1}
W_λ^s	spectral emissive flux of a source at wavelength λ, W cm^{-2} μm^{-1}
α_λ^a	absorption coefficient, cm^{-1}
α_λ^s	scattering coefficient, cm^{-1}
ϵ_λ	spectral emissivity
φ	azimuth angle
Γ_λ	transmittivity
λ	wavelength, μm
ρ	density, g cm^{-3}
σ^{sc}	scattering cross section, cm^2
τ	optical thickness
θ	polar angle
Ω	solid angle around the direction of propagation $\hat{\Omega}$
$\hat{\Omega}$	direction of propagation

RADIATIVE HEAT TRANSFER AND SELECTIVE EMISSION IN PLANAR SLAB COATINGS ON HOT EMISSIVE SUBSTRATES

INTRODUCTION

This study determines the theoretical conditions necessary for selective emission of infrared (IR) radiation from a smooth planar coating in thermal contact with a hot substrate. The standard for thermal radiation output is the blackbody, which absorbs and emits more thermal energy, either total or in an arbitrary spectral interval, than any other type of source at the same temperature. The spectral distribution curve of a blackbody provides the limiting envelope for the other type of radiators. The figure of merit as to how a real source compares to a blackbody is known as the emissivity ϵ_λ and is defined by

$$\epsilon_\lambda = \frac{W_\lambda^{source}}{W_\lambda^b}, \tag{1}$$

where W_λ^{source} denotes the spectral emissive flux (W cm^{-2} μm^{-1}) of the source at wavelength λ and W_λ^b denotes the spectral blackbody emissive flux that is given by the Planck equation [1]

$$W_\lambda^b\ (T) = \frac{c_1}{\lambda^5} \frac{1}{e^{c_2/\lambda T} - 1}, \tag{2}$$

where

h = $(6.6256 \pm 0.0005) \times 10^{-34}$ $W\ s^2$ is Planck's constant,
T is absolute temperature, K,
c = $(2.997925 \pm 0.000003) \times 10^{10}$ cm s^{-1} is the velocity of light,
c_1 = $2\pi hc^2 = (3.7415 \pm 0.0003) \times 10^4$ W cm^{-2} μm^4 is the first radiation constant,
c_2 = ch/k = $(1.43879 \pm 0.00019) \times 10^4$ μmK is the second radiation constant,
k_b = $(1.38054 \pm 0.00018) \times 10^{-23}$ $Ws\ K^{-1}$ is Boltzmann's constant, and
λ is the wavelength, μm.

From Eq. (1), three classes of sources can be distinguished by the spectral emissivity

- blackbody ϵ_λ equals 1,
- graybody ϵ_λ is a constant less than 1, and
- selective emitter ϵ_λ, a variable dependent on λ.

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Figure 1 shows the comparison between these three types of radiators. The spectral blackbody radiance, $N_\lambda^b(T)$ (Wcm^{-2} sr^{-1} μm^{-1}) is related to $W_\lambda^b(T)$ by

$$W_\lambda^b(T) = \int_{\varphi=0}^{2\pi} \int_{\mu=0}^{1} N_\lambda^b\,(T)\,\mu\,d\mu\,d\varphi = \pi N_\lambda^b(T), \tag{3}$$

where $\mu = \cos\theta$; θ is the polar angle, and φ is the azimuthal angle in spherical coordinates.

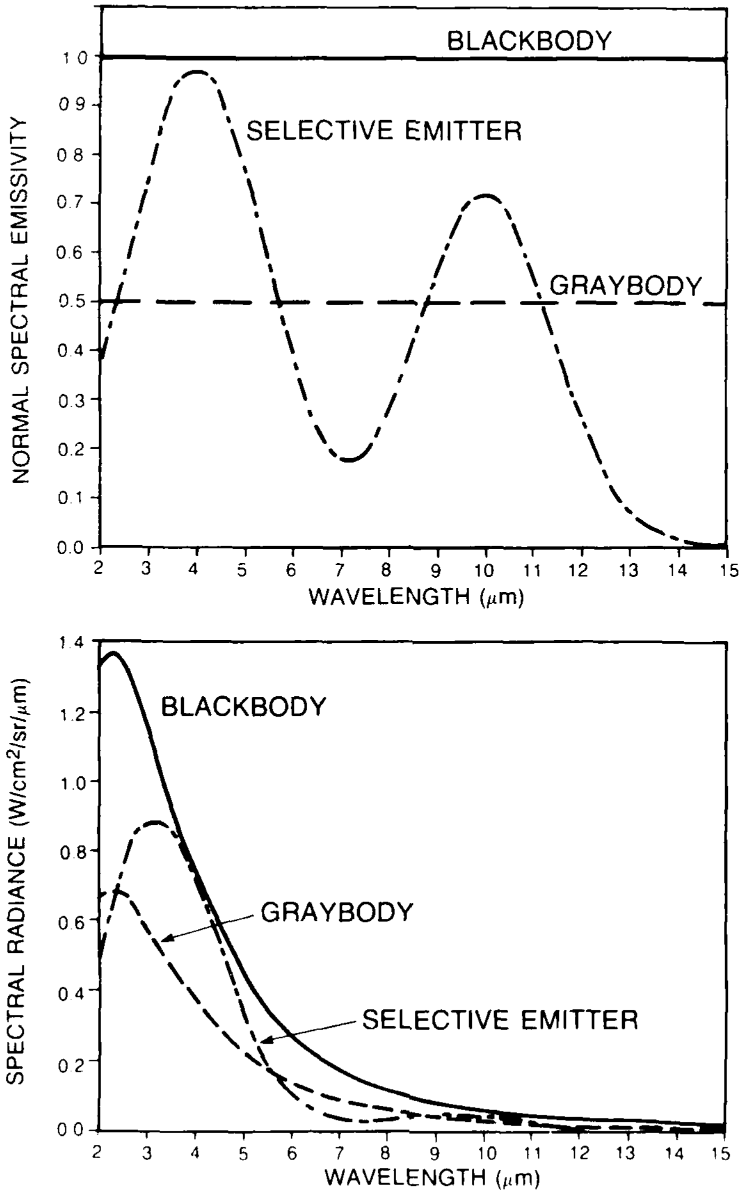


Fig. 1 — Spectral emissivity and spectral radiance of three types of radiators

A quantitative model to predict the IR output from isothermal coatings in contact with various substrates can be formulated from the solution of the radiative transfer equation. The radiative transfer equation is a first-order differential equation that governs the spatial distribution of the spectral radiance in an absorbing and emitting medium. The radiative transfer equation is well known to the heat-transfer engineer and the astrophysicist. However, in most engineering applications the net flux and temperature distribution within the system is of primary interest. A common engineering approximation is to treat the medium as a graybody and compute the net radiative flux and temperature distribution under this assumption. There has been very little formal analysis of spectral emission in such systems.

In astrophysics, it is recognized that the absorption coefficient of a stellar atmosphere varies not only with radial position but also with wavelength. Nevertheless, many of the theoretical analyses of astrophysical observations define a mean absorption coefficient and treat the stellar atmosphere as a graybody for mathematical simplicity. Since our concern is with spectral emission, the analysis formulated here must avoid the graybody approximation. In the next section a formal solution for the spectral IR output of an isothermal composite slab is obtained from the radiative transfer equation.

To make the analysis mathematically tractable we introduce a number of assumptions. The materials of the composite slab are considered homogeneous so that the attenuation of radiation by scattering can be neglected compared to absorption. In addition, the materials are considered isotropic so that the medium can be characterized by a single, uniform index of refraction. Local thermodynamic equilibrium (LTE) is assumed to exist throughout the medium so that Kirchhoff's and Planck's laws are valid. The formal solution obtained in the next section is used to investigate the effects of various input parameters on the IR output of the composite slab. The effects of coating thickness, coating absorption coefficient, substrate emissivity, and temperature are analyzed.

ENERGY TRANSFER EQUATIONS

The general equations for a conducting and radiating medium can be derived by making an energy balance on an arbitrary volume of matter. This energy balance equates the time rate of change of the energy stored in the volume element to the sum of the net heat flow into the element resulting from conduction, radiation, and heat generation within the volume element. The resulting equation for the temperature distribution is

$$\rho c_p \frac{\partial T(\vec{r}, t)}{\partial t} = -\nabla \cdot (\vec{q}_c + \vec{q}_r) + S(\vec{r}, t), \tag{4}$$

with

$$\vec{q}_c = -k \nabla T(\vec{r}, t) \text{ (Fourier Law)}, \tag{5a}$$

and

$$\vec{q}_r = \int_{\lambda=0}^{\infty} \left[\int_{4\pi} \hat{\Omega} N_\lambda(\vec{r}, \hat{\Omega}, t) d\Omega \right] d\lambda \tag{5b}$$

where

- ρ is the density,
- c_p is the specific heat at constant pressure,
- k is the thermal conductivity, and
- S is the energy generation rate per unit volume per unit time.

Equation (4) is a partial differential equation for temperature T. Once the temperature distribution is known, the heat transferred by conduction is readily determined from Eq. (5a). Since the radiation term in Eq. (5b) depends not only on the local temperature but also on the entire surrounding radiation field, the energy equation is an integrodifferential equation for the temperature distribution in the medium. The divergence of the radiative flux represents the net rate of gain for radiant energy per unit volume at any point $P(\bar{r})$ resulting from the excess of absorbed over emitted radiation. The expressions for the radiative flux and its divergence are discussed below.

Radiation Transport Equations

Consider the radiative transfer in a plane layer of material, as shown in Fig. 2, in which the temperature depends only on the coordinate in the z direction. The medium is of infinite extent in x and y directions, and the boundary conditions are such that the temperature and radiation fields do not depend on x and y. The layer is an absorbing, emitting, and scattering planar slab of thickness d that is in contact with a hot substrate that is maintained at a uniform temperature T_s. For this planar system with uniform boundary conditions, the radiative heat transfer is one-dimensional and depends on the depth only. The transfer equation (derived in Appendix A) becomes

$$\frac{dN_\lambda(s, \hat{\Omega}, t)}{ds} + (\alpha_\lambda^a + \alpha_\lambda^s)\, N_\lambda(s, \Omega, t) = \mathscr{J}_\lambda^e(s, T) + \frac{\alpha_\lambda^s}{4\pi} \int_{\Omega' = 4\pi} P(\hat{\Omega}' \cdot \hat{\Omega})\, N_\lambda(s, \hat{\Omega}')\, d\Omega', \qquad (6)$$

where the spectral radiance due to emission is $\mathscr{J}_\lambda^e(s, T)$.

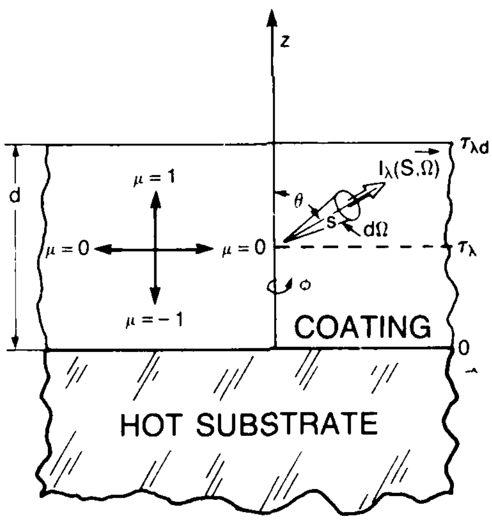


Fig. 2 — Physical model and nomenclature for a planar slab coating on a hot substrate

In Eq. (6) α_λ^a and α_λ^s are the spectral absorption and scattering coefficients, respectively, in units of cm^{-1}, s is the length measured along an arbitrary direction $\hat{\Omega}$, t is time, θ is the polar angle as depicted in Fig. 2, Ω is the solid angle around the direction of propagation $\hat{\Omega}$, and $P(\hat{\Omega}' \cdot \hat{\Omega})$ is the phase function. Equation (6) assumes that the participating medium is isotropic and homogeneous, otherwise α_λ^a and α_λ^s would be functions of direction and position. Since $z = s \cos\theta$, the directional derivative can be expressed in terms of the derivatives with respect to the space coordinate z as

$$\frac{d}{ds} = \mu \frac{\partial}{\partial z}. \qquad (7)$$

where μ is the cosine of the angle θ between the direction $\hat{\Omega}$ of the beam and the z axis. It is convenient to define the *optical thickness* τ_λ:

$$\tau_\lambda = \int_0^z (\alpha_\lambda^a + \alpha_\lambda^s)\, d\zeta' . \tag{8}$$

The independent variable s in Eq. (6) can be rewritten in terms of τ_λ.

MODEL I: Nonscattering System at Uniform Temperature T_s

For a homogeneous, isotropic coating that absorbs and emits radiation but does not scatter it ($\alpha_\lambda^s = 0$), Eq. (8) reduces to $\tau_\lambda = \alpha_\lambda^a z$ and the radiative transfer equation is written

$$\mu \frac{\partial N_\lambda(\tau_\lambda, \mu, \varphi)}{\partial \tau_\lambda} + N_\lambda(\tau_\lambda, \mu, \varphi) = \frac{\mathscr{J}_\lambda^e(\tau_\lambda, T)}{\alpha_\lambda^a}, \tag{9a}$$

with the boundary condition at $\tau_\lambda = 0$

$$N_\lambda(0, \mu) = \epsilon_\lambda^s N_\lambda^b (T_s), \tag{9b}$$

where ϵ_λ^s is the substrate emissivity and the reflected energy from interfaces at $\tau_\lambda = 0$ and $\tau_\lambda = \tau_{\lambda d}$ are assumed here to the negligible. For azimuthal symmetry, the radiance is independent of φ and Eq. (9a) becomes

$$\mu \frac{\partial N_\lambda(\tau_\lambda, \mu)}{\partial \tau_\lambda} + N_\lambda(\tau_\lambda, \mu) = \frac{\mathscr{J}_\lambda^e(\tau_\lambda, T)}{\alpha_\lambda^a} \qquad 0 \le \tau_\lambda \le \tau_{\lambda d} . \tag{10}$$

The spectral radiance $N_\lambda(\tau_\lambda, \mu)$ in the absorbing and emitting coating layer is determined from the solution of Eq. (10). Before arriving at the formal solution it is desirable to divide $N_\lambda(\tau_\lambda, \mu)$ into two contributions: the radiance directed in the positive direction ($\mu > 0$) denoted by $N_\lambda^+(\tau_\lambda, \mu)$; and the radiance directed in the negative direction ($\mu < 0$) denoted by $N_\lambda^-(\tau_\lambda, \mu)$. Equation (10) is then separated into $N_\lambda^+(\tau_\lambda, \mu)$ and $N_\lambda^-(\tau_\lambda, \mu)$ components, and the resulting equations along with the associated boundary conditions are given as

$$\mu \frac{\partial N_\lambda^+(\tau_\lambda, \mu)}{\partial \tau_\lambda} + N_\lambda^+(\tau_\lambda, \mu) = \frac{\mathscr{J}_\lambda^e(\tau_\lambda, T)}{\alpha_\lambda^a} \qquad \mu > 0, \tag{11a}$$

with

$$N_\lambda^+(0, \mu) = \epsilon_\lambda^s N_\lambda^b(T_s), \tag{11b}$$

and

$$\mu \frac{\partial N_\lambda^-(\tau_\lambda, \mu)}{\partial \tau_\lambda} + N_\lambda^-(\tau_\lambda, \mu) = \frac{\mathscr{J}_\lambda^e(\tau_\lambda, T)}{\alpha_\lambda^a} \quad \mu < 0, \tag{12a}$$

with

$$N_\lambda^-(\tau_{\lambda d}, \mu) = r_{2\lambda} N_\lambda^+(\tau_{\lambda d}, -\mu), \tag{12b}$$

where $r_{2\lambda}$ denotes the specular reflectivity off the boundary surface $\tau_\lambda = \tau_{\lambda d}$. The reflectivity, $r_{2\lambda}$, can be computed for a smooth surface from the Fresnel equations. Using an integrating factor and then combining the two terms on the left-hand side of Eq. (11a) into a total differential yields

$$\frac{d[N_\lambda^+ \exp(\tau_\lambda/\mu)]}{d\tau_\lambda} = \frac{1}{\mu\alpha_\lambda^a} \mathcal{J}_\lambda^e(\tau_\lambda, T) \exp(\tau_\lambda/\mu) \qquad \mu > 0. \tag{13a}$$

In a similar manner, Eq. (12a) becomes

$$\frac{d[N_\lambda^- \exp(\tau_\lambda/\mu)]}{d\tau_\lambda} = \frac{1}{\mu\alpha_\lambda^a} \mathcal{J}_\lambda^e(\tau_\lambda, T) \exp(\tau_\lambda/\mu) \qquad \mu < 0. \tag{13b}$$

Integration of both sides of Eqs. (13a) and (13b) with respect to τ_λ, and rearranging terms using the boundary conditions gives

$$N_\lambda^+(\tau_\lambda, \mu) = N_\lambda^+(0, \mu) \exp(-\tau_\lambda/\mu) + \frac{\exp(-\tau_\lambda/\mu)}{\mu\alpha_\lambda^a} \int_0^{\tau_\lambda} \mathcal{J}_\lambda^e(\tau'_\lambda, T) \exp(\tau'_\lambda/\mu)\, d\tau'_\lambda, \tag{14a}$$

and

$$N_\lambda^-(\tau_\lambda, \mu) = N_\lambda^-(\tau_{\lambda d}, \mu) \exp[(\tau_{\lambda d} - \tau_\lambda)/\mu] + \frac{\exp(-\tau_\lambda/\mu)}{\mu\alpha_\lambda^a} \int_{\tau_{\lambda d}}^{\tau_\lambda} \mathcal{J}_\lambda^e(\tau'_\lambda, T) \exp(\tau'_\lambda/\mu)\, d\tau'_\lambda. \tag{14b}$$

*Equations (14a) and (14b) are formal solutions for the spectral radiance in the positive and nega-*tive z directions, respectively. In general the temperature T is a function of τ_λ. However, if the coating is thin, temperature gradients can be neglected and by using the isothermal assumption, T may be approximated by T_s. With this approximation $\mathcal{J}_\lambda^e(\tau'_\lambda, T_s)$ is a constant and can be factored from inside the integral sign and Eqs. (14a) and (14b) are rewritten as

$$N_\lambda^+(\tau_\lambda, \mu) = N_\lambda^+(0, \mu) \exp(-\tau_\lambda/\mu) + \frac{\mathcal{J}_\lambda^e(T_s)}{\mu\alpha_\lambda^a} \int_0^{\tau_\lambda} \exp[(\tau'_\lambda - \tau_\lambda)/\mu)]\, d\tau'_\lambda \tag{15a}$$

$$N_\lambda^-(\tau_\lambda, \mu) = N_\lambda^-(\tau_{\lambda d}, \mu) \exp[(\tau_{\lambda d} - \tau_\lambda)/\mu] - \frac{\mathcal{J}_\lambda^e(T_s)}{\mu\alpha_\lambda^a} \int_{\tau_\lambda}^{\tau_{\lambda d}} \exp[(\tau'_\lambda - \tau_\lambda)/\mu]\, d\tau'_\lambda \tag{15b}$$

or, by replacing μ by $-\mu$, Eq. (15b) becomes

$$N_\lambda^-(\tau_\lambda, -\mu) = N_\lambda^-(\tau_{\lambda d}, -\mu) \exp[-(\tau_{\lambda d} - \tau_\lambda)/\mu] + \frac{\mathcal{J}_\lambda^e(T_s)}{\mu\alpha_\lambda^a} \int_{\tau_\lambda}^{\tau_{\lambda d}} \exp[(\tau_\lambda - \tau'_\lambda)/\mu]\, d\tau'_\lambda. \tag{16}$$

Assuming that the medium is in local thermodynamic equilibrium and that Kirchhoff's law is valid, the spectral radiance resulting from emission is given by

$$\mathcal{J}_\lambda^e(T_s) = \alpha_\lambda^a N_\lambda^b(T_s). \tag{17}$$

To determine the net spectral radiance at the optical thickness τ_λ, Eqs. (15a) and (16) must be combined as

$$N_\lambda^{\text{net}}(\tau_\lambda, \mu) = N_\lambda^+(\tau_\lambda, \mu) - N_\lambda^-(\tau_\lambda, \mu). \tag{18}$$

Integrating Eqs. (15a) and (16) with respect to τ_λ' and using the thermodynamic equilibrium condition from Eq. (17) gives

$$N_\lambda^+(\tau_\lambda, \mu) = N_\lambda^+(0, \mu) \exp(-\tau_\lambda/\mu) + N_\lambda^b(T_s)\,[1 - \exp(-\tau_\lambda/\mu)]. \tag{19a}$$

$$N_\lambda^-(\tau_\lambda, -\mu) = N_\lambda^-(\tau_{\lambda d}, -\mu) \exp[(\tau_\lambda - \tau_{\lambda d})/\mu] - N_b^\lambda(T_s)\,[\exp\{(\tau_\lambda - \tau_{\lambda d})/\mu\} - 1]. \tag{19b}$$

By using the boundary conditions for $N_\lambda^+(0, \mu)$ and $N_\lambda^-(\tau_{\lambda d}, -\mu)$ from Eqs. (11b) and (12b) the above equations become

$$N_\lambda^+(\tau_\lambda, \mu) = \epsilon_\lambda^s N_\lambda^b(T_s) \exp(-\tau_\lambda/\mu) + N_\lambda^b(T_s)\,[1 - \exp(-\tau_\lambda)/\mu] \tag{20a}$$

$$N_\lambda^-(\tau_\lambda, -\mu) = r_{2\lambda}[\epsilon_\lambda^s N_\lambda^b(T_s) \exp(-\tau_{\lambda d}/\mu) + N_\lambda^b(T_s)\,(1 - \exp(-\tau_{\lambda d}/\mu))] \exp[(\tau_\lambda - \tau_{\lambda d})/\mu]$$
$$- N_\lambda^b(T_s)[\exp\{(\tau_\lambda - \tau_{\lambda d})/\mu\} - 1]. \tag{20b}$$

Substitution of Eqs. (20a) and (20b) into Eq. 18 gives

$$N_\lambda^{\text{net}}(\tau_\lambda, \mu) = \epsilon_\lambda^s N_\lambda^b(T_s) \exp(-\tau_\lambda/\mu) + N_\lambda^b(T_s)\,[1 - \exp(-\tau_\lambda/\mu)]$$
$$- r_{2\lambda}\,[\epsilon_\lambda^s N_\lambda^b(T_s) \exp(-\tau_{\lambda d}/\mu) + N_\lambda^b(T_s)\,(1 - \exp(-\tau_{\lambda d}/\mu))] \exp[(\tau_\lambda - \tau_{\lambda d})/\mu]$$
$$+ N_\lambda^b(T_s)\,[\exp\{(\tau_\lambda - \tau_{\lambda d})/\mu\} - 1]. \tag{21}$$

The spectral radiance observed at the surface of the slab at normal incidence, i.e., $\theta = 0°$ ($\mu = 1$) is determined by setting $\tau_\lambda = \tau_{\lambda d}$ in Eq. (21) that then reduces to

$$\boxed{N_\lambda^{\text{net}}(\tau_{\lambda d}) = N_\lambda^b(T_s)[1 - r_{2\lambda}]\,[\epsilon_\lambda^s \exp(-\tau_{\lambda d}) + 1 - \exp(-\tau_{\lambda d})].} \tag{22}$$

Equation (22) is the central result of our analysis. This is the general solution for the radiance emitted from an absorbing, emitting, nonscattering planar slab coating of thickness d on a hot substrate of emissivity ϵ_λ^s. The physical significance of the various terms in Eq. (22) are as follows: The first term on the right-hand side is the contribution from the hot substrate attenuated by the optical thickness $\tau_{\lambda d}$; the second term is the contribution to the radiance from the emission along the path $\tau = 0$ to $\tau = \tau_{\lambda d}$; the factor $(1 - r_{2\lambda})$ accounts for the specular reflectivity at the boundary surface between the coating and the outside medium. Consideration shall now be given to some special cases of the input parameters.

Case I: Substrate Is a Blackbody ($\epsilon_\lambda^s = 1$ and $r_{2\lambda} = 0$).

Equation (22) reduces to

$$N_\lambda^{\text{net}}(\tau_{\lambda d}) = N_\lambda^b(T_s)\,[1] = N_\lambda^b(T_s). \tag{23}$$

This leads to Conclusion I:

If a coating of a selective emitter is applied to a blackbody substrate and forms a non-scattering, absorbing, and emitting layer in thermodynamic equilibrium with the substrate, then the composite system can only emit as a **BLACKBODY**.

Case II: Role of Optical Thickness

The optical thickness $\tau_{\lambda d}$ was defined by Eq. (8) and related the absorption coefficient α_λ^a and the geometric thickness d of the layer. In an optically thick regime, the quantity $\tau_{\lambda d}$ approaches infinity. The exponential terms in Eq. (22) approach zero and when $r_{2\lambda} = 0$, this equation reduces to

$$N_\lambda^{\text{net}}(\tau_{\lambda d}) = N_\lambda^b(T_s)\,[1] = N_\lambda^b(T_s). \tag{24}$$

This result leads to Conclusion II:

In an optically thick limit the composite system described by Fig. 2 can emit only as a **BLACKBODY**.

This conclusion is independent of the nature of the substrate and the selective emission characteristics of the coating. It is also valid for emission from hot gases such as those found in IR flares [2]. The addition of selective emitters to a hot flare plume cannot change the spectral IR output if the plume is optically thick. Figure 3 shows an example of how the optical thickness effects the IR radiance of sheets of ordinary window glass of various thicknesses at 1000°C. Note that the spectral emissive power approaches that of a blackbody as the sheet thickness x goes to infinity for nonzero spectral absorption coefficients. The reflectivity factor in Eq. (22) causes the emissivity to differ from 1.0 as the glass thickness approaches infinity.

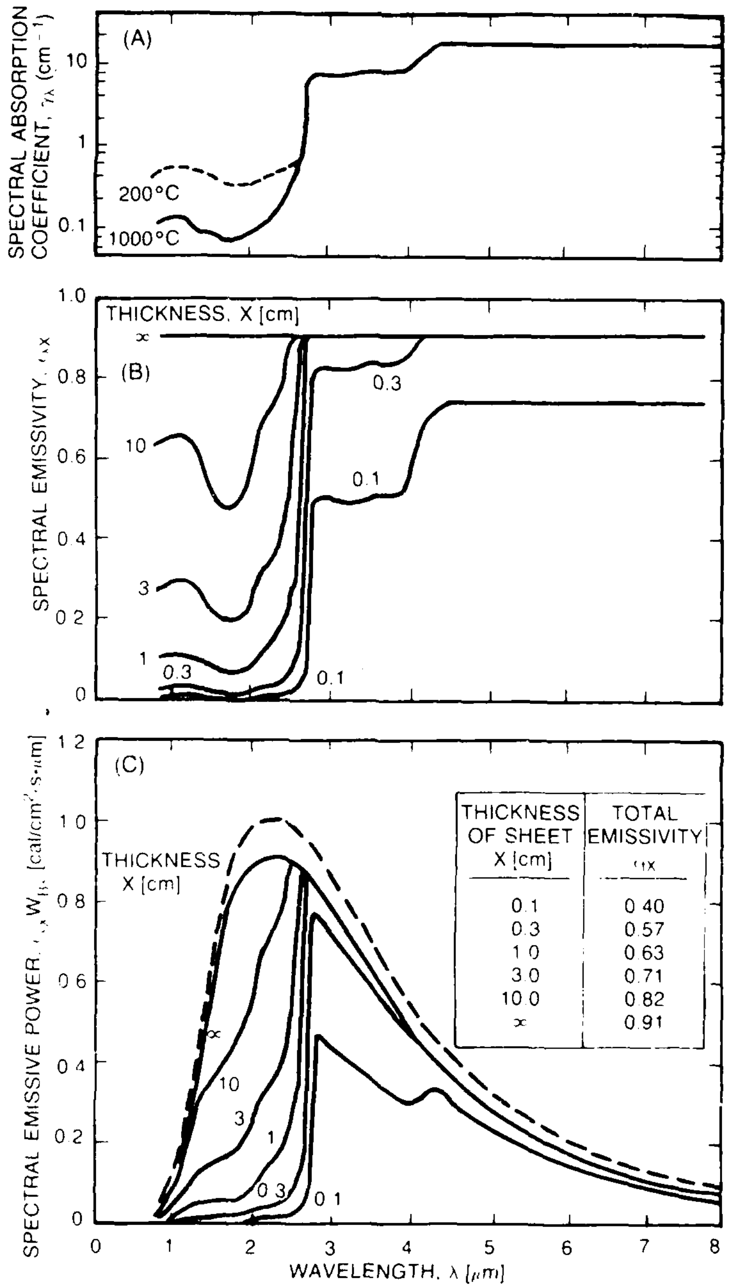
When $\alpha_\lambda^a = 0$, the optical thickness τ_λ is 0, the exponential terms approach 1 in Eq. (22), and when $r_{2\lambda} = 0$, Eq. (22) reduces to

$$N_\lambda^{\text{net}}(\tau_{\lambda d}) = N_{\lambda_b}(T_s)\epsilon_\lambda^s. \tag{25}$$

Under these circumstances the spectral radiance is determined entirely by the substrate emissivity and temperature.

The *spectral transmissivity* Γ_λ measured in the direction of propagation is defined as

$$\Gamma_\lambda = e^{-\tau_\lambda} = e^{-\alpha_\lambda^a z}. \tag{26}$$



THICKNESS OF SHEET X [cm]	TOTAL EMISSIVITY ϵ_X
0.1	0.40
0.3	0.57
1.0	0.63
3.0	0.71
10.0	0.82
∞	0.91

Fig. 3 — Emissivity of sheets of window glass at 1000 C
[data taken from Ref. 3]

The conservation of energy within the coating is expressed by

$$a_\lambda + \Gamma_\lambda + r_\lambda = 1, \tag{27}$$

where a_λ is the spectral absorptivitv and r_λ is the spectral reflectivity.

In the present analysis, the spectral reflectivity is assumed to be negligible: i.e., $r_\lambda \Rightarrow 0$ and Kirchhoff's law is assumed to be applicable, which implies that $a_\lambda = \epsilon_\lambda$. Equation (27) then can be written in terms of the emissivity of the coating layer of thickness d as

$$a_\lambda^c = 1 - e^{-\alpha_\lambda^a d} = 1 - \exp(-\tau_{\lambda d}) = \epsilon_\lambda^c. \tag{28}$$

Substituting Eq. (28) into Eq. (22) yields the general solution

$$N_\lambda^{\text{net}}(\tau_{\lambda d}) = N_\lambda^b(T_s)\,[\epsilon_\lambda^s \exp(-\tau_{\lambda d}) + \epsilon_\lambda^c]. \tag{29}$$

In the optically thin regime, $\tau_{\lambda d}$ approaches 0, and Eq. (29) is approximated by

$$N_\lambda^{\text{net}}(\tau_{\lambda d}) = N_\lambda^b(T_s)\,[\epsilon_\lambda^s + \epsilon_\lambda^c - \epsilon_\lambda^s\epsilon_\lambda^c], \tag{30}$$

where ϵ_λ^c is the emissivity of the coating. This equation expresses the net spectral radiance in terms of measurable or assumed spectral emissivities.

Case III: Opaque Substrate of Negligible Emissivity ($\epsilon_\lambda^s << 1$)

If the substrate emissivity ϵ_λ^s in Eq. (29) is small, the intensity equation reduces to the approximate form

$$N_\lambda^{\text{net}}(\tau_{\lambda d}) = N_\lambda^b(T_s)\,[\epsilon_\lambda^c]. \tag{31}$$

Figure 4 shows the measured selective emission of a finite thickness of silicon dioxide ($\epsilon_\lambda^s \equiv 0$). Equation (31) reveals that if a selective emitter is applied to a substrate of much lower spectral emissivity (e.g., a metal), the radiated intensity from the composite system is dominated by the emissivity of the coating layer.

This leads to Conclusion III:

> *If a coating of a selective emitter is applied to a metallic (low emissivity, i.e., IR blocker) substrate and forms a nonscattering, absorbing, emitting layer in thermodynamic equilibrium with the substrate, the spectral radiance of the composite system will be dominated by the* **EMISSIVE PROPERTIES OF THE COATING**.

As an example of this case, consider a hot steel substrate at 1273 K (1000°C) coated with a layer of silicon dioxide in thermal equilibrium. Figure 5 shows the normal spectral emissivity for these materials at the above temperature. The radiance for a blackbody and the steel and silicon dioxide composite in the 2 to 15-μm region at this temperature can be compared by using Eq. (30). Figure 6 shows the results, which demonstrate the selective IR emission for this system.

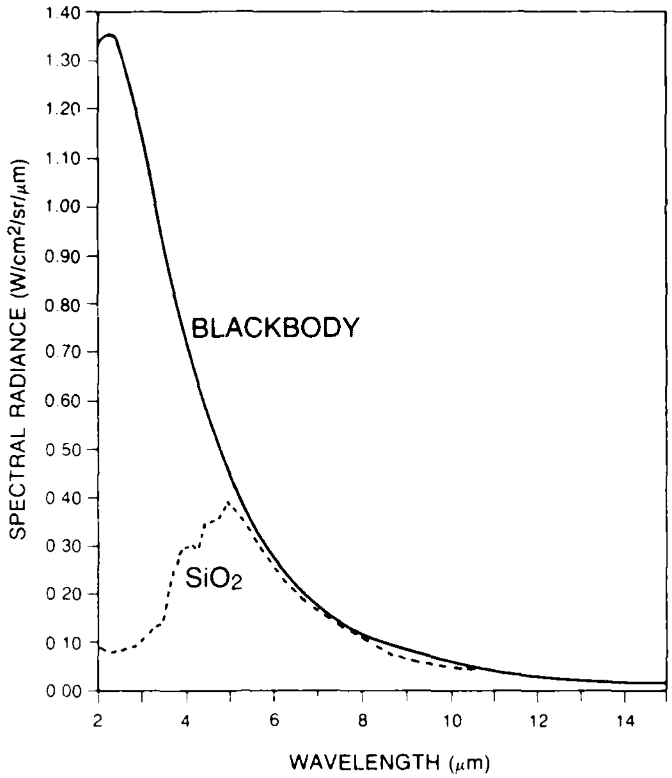


Fig. 4 — Comparison of the IR output of SiO_2 (110 mils thick) and a blackbody at 1000°C

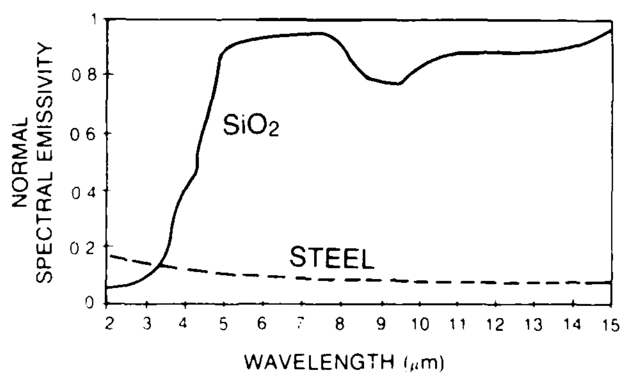


Fig. 5 — Normal spectral emissivity of SiO_2 (110 mils thick) and steel at 1000°C [SiO_2 data taken from Ref. 4 and steel data taken from Ref. 5]

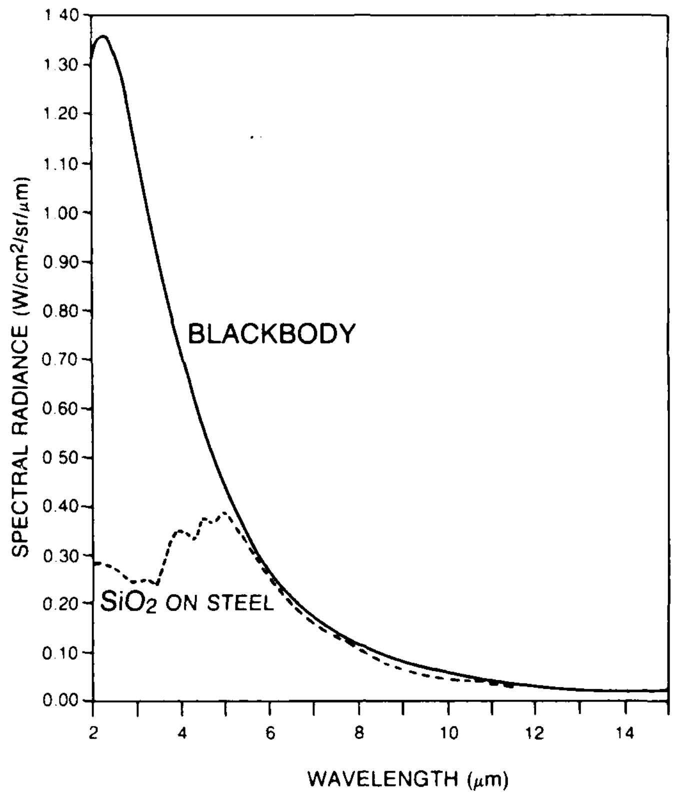


Fig. 6 — Comparison of the IR output of composite steel and SiO_2 coating (110 mils thick) on a blackbody substrate vs a blackbody at 1000°C

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REFERENCES

1. R.D. Hudson, *Infrared Systems Engineering* (Wiley, New York, 1969).

2. W. Finkelnburg, "Conditions for Blackbody Radiation of Gases," *J. Opt. Soc. Am.* **39**(2), 185-186 (1949).

3. R. Gardon, "The Emissivity of Transparent Materials," *J. Am. Ceram. Soc.* **39**(8), 278-287 (1956).

4. E. Schatz and L. McCandless, "Research for Low and High Emittance Coatings," ASD TR 62-443, May 1962, p. 64.

5. M.A. Bramson, *Infrared Radiation* (Plenum Press, New York, 1968), pp. 126-134.

Appendix A

DERIVATION OF THE RADIATIVE TRANSFER EQUATION

The propagation of radiation at any point in a medium cannot be represented by a single vector as in the case of heat flow by conduction. To specify the radiation incident at a given point within the medium, one must know the radiation from all directions because radiation beams are independent of one another. The amount of radiation energy transmitted by the ray in any given direction per unit time is the spectral radiance. To define the quantity we consider an element of surface area dA located about the space coordinate $\bar{r}$ as illustrated in Fig. A1. Let $\hat{n}$ be a normal unit-direction vector to dA and let dE_λ denote the amount of radiative energy in the wavelength interval λ to $\lambda + d\lambda$, confined to an element of solid angle $d\Omega$ around the direction of propagation $\hat{\Omega}$ streaming through the elemental surface dA during the time interval between t and $t + dt$. Let θ be the polar angle between $\hat{n}$ and the direction of propagation $\hat{\Omega}$. The *spectral radiance* N_λ is defined as

$$N_\lambda = \frac{dE_\lambda}{dA \cos\theta \, dt \, d\Omega \, d\lambda}. \tag{A1}$$

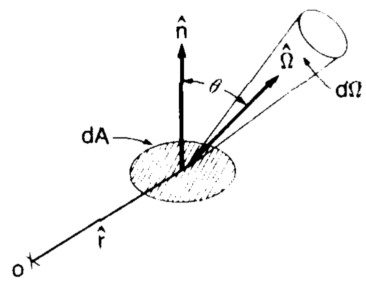

Fig. A1 — Symbols for the definition of radiance

The spectral radiance in a given direction in a nonattenuating and nonemitting medium with constant properties is independent of position along that direction. This invariance of spectral radiance when no attenuation or emission is present provides a convenient way of specifying the magnitude of any attenuation or emission as these effects are given directly by the change in radiance with distance traveled through the medium. In the engineering community, the quantity defined by Eq. (A1) is often called the *spectral intensity*.

Radiation traveling along a path is attenuated by absorption and scattering and enhanced by emission (both spontaneous and induced) and radiation scattered in the path direction from other directions within the medium. The processes of absorption, emission, and scattering can be written as an energy balance over a small volume element of the medium. Consider an elementary cylinder, the axis of which is directed along a given direction of propagation $\hat{\Omega}$ ($\cos\theta = 1$). Let the area of the cylinder's base be dA, and let the cylinder's height be ds. The spectral radiance where the ray enters the cylinder is denoted by N_λ, and the spectral radiance leaving the cylinder is denoted by $N_\lambda + dN_\lambda$. If the medium within the cylinder interacts with the radiation, the net change in the spectral radiance is dN_λ. By equation (A1), the quantity

$$dN_\lambda \, dA \, dt \, d\Omega \, d\lambda , \tag{A2}$$

represents the difference in the radiative energy crossing the surface dA at $s + ds$ and s in the time interval dt about t and in the wavelength interval $d\lambda$ about λ, and contained in an element of solid angle $d\Omega$ about the direction $\hat{\Omega}$. Let G_λ represent the net gain of radiative energy by the beam in the volume element $dAds$ per unit volume, per unit time about t, per unit wavelength about λ, and per unit solid angle about the direction $\hat{\Omega}$. Then the quantity

$$G_\lambda dAdtd\Omega d\lambda ds , \tag{A3}$$

represents the net gain of radiative energy by the beam contained in an element of solid angle $d\Omega$ about the direction $\hat{\Omega}$, in the time interval dt, in the wavelength interval $d\lambda$, and in the cylindrical volume element $dAds$. By equating the quantities (A2) and (A3), we obtain

$$\frac{dN_\lambda}{ds} = G_\lambda .$$

The spatial derivative in Eq. (A4) assumes that the observer is moving with the radiation beam at a speed c (Lagrangian frame). The distance ds traversed by the beam in time dt is given by

$$ds = cdt . \tag{A5}$$

To compute the derivative with respect to space in a fixed coordinate system (Eulerian frame), one must use the substantial derivative and Eq. (A4) becomes

$$\frac{1}{c}\frac{\partial N_\lambda}{\partial t} + \hat{\Omega} \cdot \nabla \vec{N}_\lambda = G_\lambda . \tag{A6}$$

If the direction of propagation is along the path s, Eq. (A6) can be written as

$$\frac{1}{c}\frac{\partial N_\lambda}{\partial t} + \frac{\partial N_\lambda}{\partial s} = G_\lambda . \tag{A7}$$

In most applications, the first term in this equation can be neglected because of the large magnitude of the speed of propagation c. We now derive explicit expressions for the various components of G_λ:

$$G_\lambda = G_\lambda^a + G_\lambda^{os} + G_\lambda^{Is} + G_\lambda^{EM} , \tag{A8}$$

where G_λ^a is absorption, G_λ^{os} is out-scattering, G_λ^{Is} is in-scattering , and G_λ^{EM} is emission.

Consider radiation of radiance $N(s, \hat{\Omega})$ traveling in an absorbing, emitting, and scattering medium. As the radiation passes through the cylindrical volume element, it is attenuated by absorption. The decrease in energy per unit volume, per unit time about t, per unit wavelength about λ, and per unit solid angle about the direction $\hat{\Omega}$ is given by the phenomenological Bouger-Lambert Law

$$G_\lambda^a = - \alpha_\lambda^a N_\lambda . \tag{A9}$$

The spectral absorption coefficient α_λ^a is a physical property of the medium and has the units of reciprocal length. In general, it is a function of temperature, pressure, material composition, and radiation wavelength. The absorption process considered in Eq. (A9) includes both "true absorption" and stimulated emission because both processes are proportional to the radiance of the incident radiation. The net absorption of the medium may be positive or negative, depending on whether "true absorption" or stimulated emission dominates.

The extent of scattering within the medium depends on the number density of scattering particles (N_p) within the medium and the scattering cross section (σ_λ^s) of an individual scattering particle. The cross section for scattering is the apparent area that a particle presents to an incident beam insofar as the particle's ability to deflect radiation from the beam is concerned. The scattering cross section depends on the particle size, geometry, material composition, radiation wavelength, and polarization. The decrease in spectral radiance per unit time, volume, solid angle, and wavelength resulting from the scattering out of the incident beam direction is given by

$$G_\lambda^{so} = -\sigma_\lambda^s N_p N_\lambda = -\alpha_\lambda^s N_\lambda , \tag{A10}$$

where α_λ^s denotes the spectral scattering coefficient. The scattering coefficient can be regarded as the reciprocal of the mean free path that the radiation traverses before being scattered.

Since radiation is scattered from paths adjacent to the volume element under consideration, the scattering process can also enhance the spectral radiance along the distance ds. To calculate the enhancement from this incoming scattering, the directional distribution of the scattered radiation is required. This distribution is described by an angularly dependent phase function $P_\lambda(\hat{\Omega}' \cdot \hat{\Omega})$. Physically, the phase function is interpreted as the scattered radiance in a particular direction divided by the radiance that would be scattered in that direction if the scattering were isotropic. If the scattering particles in the medium are modeled as isotropic spheres and the medium has no preferential direction for scattering, then the phase function depends only on the angle between the incident direction $\hat{\Omega}$ and the scattered direction $\hat{\Omega}'$. Mathematically, the phase function can be related to a probability density. The quantity defined by

$$\frac{1}{4\pi} P_\lambda(\hat{\Omega}' \cdot \hat{\Omega})\, d\Omega' , \tag{A11}$$

represents the probability that an incident beam of radiation traveling in direction $\hat{\Omega}$ will be scattered into an element of solid angle $d\Omega'$ about the direction $\hat{\Omega}'$. The scattering of the incident radiation, $N_\lambda(\bar{r}, \hat{\Omega})$, by the medium per unit time, per unit volume, per unit wavelength into an element of solid angle $d\Omega'$ about the direction $\hat{\Omega}'$ is given by

$$\alpha_\lambda^s N_\lambda(s, \hat{\Omega})\, d\Omega\, \frac{1}{4\pi} P_\lambda(\hat{\Omega}' \cdot \hat{\Omega})\, d\Omega' . \tag{A12}$$

In general, the scattered radiation will be incident on the volume element from all directions. The integration of Eq. (A12) over all solid angles of incidence yields

$$G_\lambda^{is} = \frac{\alpha_\lambda^s}{4\pi} \int P_\lambda(\hat{\Omega}' \cdot \hat{\Omega})\, N_\lambda(s, \hat{\Omega})\, d\Omega , \tag{A13}$$

which represents the scattering of radiation incident on the volume element from all directions into the solid angle $d\Omega'$ about the direction $\hat{\Omega}'$ per unit time, per unit volume, and per unit wavelength.

If the medium is in local thermodynamic equilibrium, spontaneous emission along path length ds enhances the radiance in the s direction by

$$G_\lambda^{EM} = \alpha_\lambda^a N_\lambda^b , \tag{A14}$$

where N_λ^b is Planck's function. Equation (A14) is a mathematical statement of Kirchhoff's law. The assumption of local thermodynamic equilibrium simply means that the matter contained in the cylindrical volume is in thermodynamic equilibrium with itself but not necessarily with the radiation field [A1].

Given these explicit expressions for the components of G_λ, the radiative transfer equation immediately follows from Eqs. (A4) and (A8):

$$\frac{dN_\lambda}{ds} = -(\alpha_\lambda^s + \alpha_\lambda^a)\, N_\lambda + \frac{\alpha_\lambda^s}{4\pi} \int P_\lambda\,(\hat{\Omega}' \cdot \hat{\Omega})\, N_\lambda(s, \hat{\Omega})\, d\Omega + \alpha_\lambda^a N_\lambda^b , \tag{A15}$$

or

$$\frac{dN_\lambda(s, \hat{\Omega}, t)}{ds} + (\alpha_\lambda^a + \alpha_\lambda^s) N_\lambda(s, \Omega, t) = \alpha_\lambda^a N_\lambda^b(s, T) + \frac{\alpha_\lambda^s}{4\pi} \int_{\Omega' = 4\pi} P_\lambda(\hat{\Omega}' \cdot \hat{\Omega}) N_\lambda(s, \hat{\Omega}') d\Omega' . \tag{A16}$$

For further details see Ref. A2.

REFERENCES

A1. H.P. Baltes, "On the Validity of Kirchhoff's Law of Heat Radiation for a Body in a Nonequilibruim Environment," in *Progress in Optics* XIII, E. Wolf, ed. (North-Holland, Amsterdam, 1976).

A2. M.N. Ozisik, *Radiative Transfer and Interactions with Conduction and Convection* (Wiley, New York, 1973).

Appendix B

AVERAGE (HEMISPHERICAL) SPECTRAL RADIANCE CALCULATION

The average spectral radiance for azimuthally symmetric radiation is defined as

$$\bar{G}_\lambda(\tau_\lambda) = \frac{\int_0^{2\pi}\int_{-1}^{1} N_\lambda(\tau_\lambda, \mu)\, d\mu\, d\varphi}{\int_0^{2\pi}\int_{-1}^{1} d\mu\, d\varphi} \tag{B1}$$

which in the present case of radiances in the forward and backward directions becomes

$$\bar{G}_\lambda(\tau_\lambda) = \frac{\int_0^{1} N_\lambda^+(\tau_\lambda, \mu)\, d\mu}{\int_0^{1} d\mu} - \frac{\int_{-1}^{0} N_\lambda^-(\tau_\lambda, \mu)\, d\mu}{\int_{-1}^{0} d\mu}. \tag{B2}$$

Substitution of Eq. (15a), and (16) into Eq. (B2) yields the formal solution for the spectral radiance as

$$\begin{aligned} \bar{G}(\tau_\lambda) = & \int_0^1 N_\lambda^+(0, \mu) \exp(-\tau_\lambda/\mu)\, d\mu \\ & + \int_0^1 \frac{f_\lambda^s(T_s)}{\mu\alpha_\lambda^a} \int_0^{\tau_\lambda} \exp[(\tau_\lambda' - \tau_\lambda)/\mu]\, d\tau_\lambda'\, d\mu \\ & - \int_0^1 N_\lambda^-(\tau_{\lambda d}, -\mu) \exp[(\tau_\lambda - \tau_{\lambda d})/\mu] d\mu \\ & + \int_0^1 \frac{f_\lambda^s(T_s)}{\mu\alpha_\lambda^a} \int_{\tau_\lambda}^{\tau_{\lambda d}} \exp[(\tau_\lambda - \tau_\lambda')/\mu]\, d\tau_\lambda'\, d\mu. \end{aligned} \tag{B3}$$

By using the boundary conditions for $N_\lambda^+(0, \mu)$, and $N_\lambda^-(\tau_{\lambda d}, \mu)$ from Eqs. (11b) and (12b) and the thermodynamic equilibrium condition from Eq. (17) the above equation becomes

$$\begin{aligned} \bar{G}_\lambda(\tau_\lambda) = & \epsilon_\lambda^s N_\lambda^b(T_s) \int_0^1 \exp(-\tau_\lambda/\mu)\, d\mu \\ & + N_\lambda^b(T_s) \int_0^1 \int_0^{\tau_\lambda} \frac{1}{\mu} \exp[(\tau_\lambda' - \tau_\lambda)/\mu]\, d\tau_\lambda'\, d\mu \\ & - \int_0^1 r_\lambda^s \{\epsilon_\lambda^s N_\lambda^b(T_s) \exp[-\tau_{\lambda d}/\mu] + N_\lambda^b(T_s)[1 - \exp(-\tau_{\lambda d}/\mu)]\} [\exp[(\tau_\lambda - \tau_{\lambda d})/\mu]]\, d\mu \\ & + N_\lambda^b(T_s) \int_0^1 \int_{\tau_\lambda}^{\tau_{\lambda d}} \frac{1}{\mu} \exp[(\tau_\lambda - \tau_\lambda')/\mu]\, d\tau_\lambda'\, d\mu \end{aligned} \tag{B4}$$

By using the exponential integral function defined as

$$E_n(z) = \int_0^1 \eta^{n-2} e^{-z/\eta}\, d\eta , \tag{B5}$$

Eq. (B4) is written as

$$\begin{aligned}\bar{G}_\lambda(\tau_\lambda) = {} & \epsilon_\lambda^s N_\lambda^b(T_s) E_2(\tau_\lambda) + N_\lambda^b \int_0^{\tau_\lambda} E_1(\tau_\lambda - \tau'_\lambda)\, d\tau' \\ & - r_{2\lambda} N_\lambda^b(T_s)[\epsilon_\lambda^s E_2(\tau_\lambda) + E_2(\tau_{\lambda d} - \tau_\lambda) + E_2(2\tau_{\lambda d} - \tau_\lambda)] \\ & + N_\lambda^b(T_s) \int_{\tau_\lambda}^{\tau_{\lambda d}} E_1(\tau'_\lambda - \tau_\lambda)\, d\tau' .\end{aligned}$$

The two integral terms in Eq. (B6) can be reduced to a simpler form by using an identity of the exponential integral as

$$\begin{aligned} & N_\lambda^b(T_s) \left[\int_0^{\tau_\lambda} E_1(\tau'_\lambda - \tau_\lambda)\, d\tau'_\lambda + \int_{\tau_\lambda}^{\tau_{\lambda d}} E_1(\tau'_\lambda - \tau_\lambda)\, d\tau'_\lambda \right] \\ & = N_\lambda^b(T_s)[2 - E_2(\tau_\lambda) - E_2(\tau_{\lambda d} - \tau_\lambda)] . \end{aligned} \tag{B7}$$

Substitution of Eq. (B7) into Eq. (B6) gives the average spectral radiance as

$$\begin{aligned}\bar{G}_\lambda(\tau_\lambda) = {} & N_\lambda^b(T_s)\, \{\epsilon_\lambda^s E_2(\tau_\lambda) + 2 - E_2(\tau_\lambda) \\ & - E_2(\tau_{\lambda d} - \tau_\lambda) - r_{2\lambda}[\epsilon_\lambda^s E_2(\tau_\lambda) \\ & + E_2(\tau_{\lambda d} - \tau_\lambda) - E_2(2\tau_{\lambda d} - \tau_\lambda)]\} .\end{aligned} \tag{B8}$$

The spectral radiance observed at the surface of the slab is determined by setting $\tau_\lambda = \tau_{\lambda d}$ in Eq. (B8), which then becomes

$$\bar{G}_\lambda(\tau_{\lambda d}) = N_\lambda^b(T_s)\, [\epsilon_\lambda^s E_2(\tau_{\lambda d}) + 1 - E_2(\tau_{\lambda d}) - r_{2\lambda}[\epsilon_\lambda^s E_2(\tau_{\lambda d}) + 1 - E_2(\tau_{\lambda d})]] \tag{B9a}$$

or

$$\bar{G}_\lambda(\tau_{\lambda d}) = N_\lambda^b(T_s)\, [1 - r_{2\lambda}]\, [1 + E_2(\tau_{\lambda d})\, (\epsilon_\lambda^s - 1)] . \tag{B9b}$$

Under these circumstances the spectral radiance is determined entirely by the substrate emissivity and temperature. When τ_λ is small, the exponential function $E_2(\tau_\lambda)$ can be expanded into a series expansion approximation as

$$E_2(\tau_\lambda) = 1 - \tau_\lambda + \gamma\tau_\lambda + 0(\tau_\lambda^2), \tag{B10}$$

where $\gamma = 0.577216$ (Euler's constant), and $0(\tau_\lambda^2)$ are higher order terms.

Substitution of Eq. (B10) into Eq. (B9) yields

$$\bar{G}_\lambda(\tau_{\lambda d}) = N_\lambda^b(T_s)\,[1 - r_{2\lambda}]\,[1 + (1 - \tau_\lambda + \gamma\tau_\lambda)\,(\epsilon_\lambda^s - 1)], \tag{B11}$$

which can be reduced to

$$\bar{G}_\lambda(\tau_{\lambda d}) = N_\lambda^b(T_s)\,[1 - r_{2\lambda}]\,[\epsilon_\lambda^s(1 - \gamma'\tau_\lambda) + \gamma'\tau_\lambda], \tag{B12}$$

where $\gamma' = 1 - \gamma$.

By using $\tau_\lambda = \epsilon_\lambda^c$ from Eq. (29), the average spectral radiance in the above equation becomes

$$\bar{G}_\lambda(\tau_{\lambda d}) = N_\lambda^b(T_s)\,[1 - r_{2\lambda}]\,[\epsilon_\lambda^s(1 - \gamma'\epsilon_\lambda^c) + \gamma'\epsilon_\lambda^c], \tag{B13}$$

or

$$\bar{G}_\lambda(\tau_{\lambda d}) = N_\lambda^b(T_s)\,[1 - r_{2\lambda}]\,[\epsilon_\lambda^s + \bar{\epsilon}_\lambda^c - \epsilon_\lambda^s\,\bar{\epsilon}_\lambda^c], \tag{B14}$$

where $\bar{\epsilon}_\lambda^c = \gamma'\epsilon_\lambda^c$.

Upon comparison of net radiance normal to the surface of the slab (Eq. 30) and the average radiance over the hemisphere (Eq. B14) with $r_{2\lambda} = 0$, it is seen that the effect of averaging comes in by replacing the emissivity of the coating layer by hemispherical emissivity.

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